

Arithmetik – Lineare Gleichungen mit drei Variablen

Lösungsblatt

Lösen Sie folgende Gleichungen über die Grundmenge $G = \mathbb{R}$!

$$\begin{aligned} I: & 5x - 4y + 3z = 4 \\ II: & 4x + 6y - z = 9 \\ III: & 3x - 2y + 4z = 5 \end{aligned}$$

$$\begin{aligned} I: & 5x - 4y + 3z = 4 \\ II: & 4x + 6y - z = 9 \quad | \cdot 3 \\ I: & 5x - 4y + 3z = 4 \\ II: & 12x + 18y - 3z = 27 \\ \Gamma: & 17x + 14y = 31 \\ II: & 4x + 6y - z = 9 \quad | \cdot 4 \\ III: & 3x - 2y + 4z = 5 \\ II: & 16x + 24y - 4z = 36 \\ III: & 3x - 2y + 4z = 5 \\ \Gamma: & 19x + 22y = 41 \end{aligned}$$

$$\begin{aligned} \Gamma: & +17x + 14y = 31 \quad | \cdot (-22) \\ \Gamma: & 19x + 22y = 41 \quad | \cdot (14) \\ I': & -374x - 308y = -682 \\ \Gamma: & +266x + 308y = +574 \\ & -108x = -108 \quad | : (-108) \\ x &= +1 \end{aligned}$$

$$\begin{aligned} \Gamma: & 19x + 22y = 41 \\ 19 \cdot 1 + 22y &= +41 \\ 19 + 22y &= +41 \quad | -19 \\ +22y &= +22 \quad | : 22 \\ y &= +1 \end{aligned}$$

$$\begin{aligned} I: & 5x - 4y + 3z = 4 \\ 5 \cdot 1 - 4 \cdot 1 + 3z &= 4 \\ 5 - 4 + 3z &= 4 \quad | -1 \\ +3z &= +3 \quad | : 3 \\ z &= +1 \\ L &= \{+1, +1, +1\} \end{aligned}$$

$$\begin{aligned} I: & 4x - 3y - 3z = -3 \\ II: & 8x + 5y + 6z = +53 \\ III: & 2x + 4y - 5z = -10 \end{aligned}$$

$$\begin{aligned} I: & 4x - 3y - 3z = -3 \quad | \cdot 2 \\ II: & 8x + 5y + 6z = +53 \\ I: & 8x - 6y - 6z = -6 \\ II: & 8x + 5y + 6z = +53 \\ \Gamma: & +16x - y = +47 \\ II: & 8x + 5y + 6z = +53 \quad | \cdot 5 \\ III: & 2x + 4y - 5z = -10 \quad | \cdot 6 \\ II: & +40x + 25y + 30z = +265 \\ III: & +12x + 24y - 30z = -60 \\ \Gamma: & +52x + 49y = +205 \\ \Gamma: & +16x - y = +47 \quad | \cdot 49 \\ \Gamma: & +52x + 49y = +205 \\ I': & +784x - 49y = +2303 \\ \Gamma: & +52x + 49y = +205 \\ & +836x = +2308 \quad | : 836 \\ x &= +3 \end{aligned}$$

$$\begin{aligned} \Gamma: & +16x - y = +47 \\ 16 \cdot 3 - y &= +47 \\ 48 - y &= +47 \quad | -48 \\ -y &= -1 \quad | \cdot (-1) \\ y &= +1 \end{aligned}$$

$$\begin{aligned} I: & 4x - 3y - 3z = -3 \\ 4 \cdot 3 - 3 \cdot 1 - 3z &= -3 \\ 12 - 3 - 3z &= -3 \quad | -9 \\ -3z &= -12 \quad | : (-3) \\ z &= +4 \\ L &= \{+3, +1, +4\} \end{aligned}$$