Funktionen – Anwendung der Integralrechnen – Volumen von Rotationskörpern

Arbeitsblatt 2

Berechnen Sie das Volumen des Rotationskörpers, der bei Rotation der Ellipse

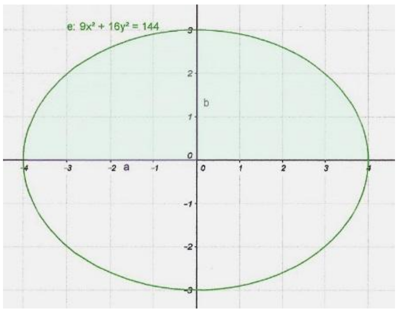
ϵ: $9x^2 + 16y^2 = 144$ a *) um die x-Achse   b *) um die y-Achse   entsteht!

ϵ: $b^2x^2 + a^2y^2 = a^2 \,.\, b^2$

$\underline{\epsilon\text{:}\ \ 9x^2 + 16y^2 = 144}$ $\rightarrow$ $a^2 = 16$; $a = 4$;

$\rightarrow$ $b^2 = 9$; $b = 3$; → siehe Skizze!

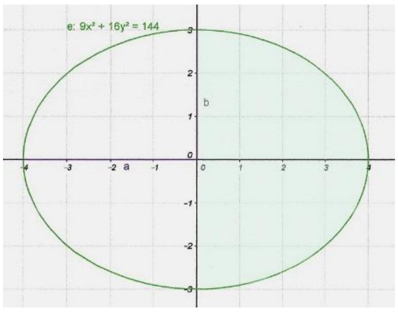
$x^2 = 16 - \frac{16}{9}y^2$; $y^2 = 9 - \frac{9}{16}x^2$;



<u>Rotation um die x–Achse</u>:

$V_x = \pi \,.\, \int_{-a}^{+a} y^2 \,.\, dx =$

$= \underline{48 \,.\, \pi}$



<u>Rotation um die y–Achse:</u>

$V_y = \pi \,.\, \int_{-b}^{+b} x^2 \,.\, dy =$

$= \underline{64 \,.\, \pi}$

$\underline{V_x = 48 \,.\, \pi}$ VE $\underline{V_y = 64 \,.\, \pi}$ VE $V_x : V_y = 48 : 64 = 3 : 4$

Ebenso: ϵ: $4x^2 + 25y^2 = 100$; $V_x = ?$; $V_y = ?$;

ϵ: $b^2x^2 + a^2y^2 = a^2 \,.\, b^2$ $\rightarrow$ $a^2 = 25$; $a = 5$; $\rightarrow$ $b^2 = 4$; $b = 2$;

$\underline{\epsilon\text{:}\ \ 4x^2 + 25y^2 = 100}$ $\rightarrow$ $x^2 = 25 - \frac{25}{4}y^2$; $\rightarrow$ $y^2 = 4 - \frac{4}{25}x^2$;

<u>Rotation um die x–Achse</u>:

$V_x = \pi \,.\, \int_{-a}^{+a} y^2 \,.\, dx =$

$= \frac{\mathbf{80}}{\mathbf{3}} \,.\, \boldsymbol{\pi}$

<u>Rotation um die y–Achse:</u>

$V_y = \pi \,.\, \int_{-b}^{+b} x^2 \,.\, dy =$

$= \frac{\mathbf{200}}{\mathbf{3}} \boldsymbol{.\pi}$

$V_x = \left(\frac{\mathbf{80}}{\mathbf{3}}\right)\boldsymbol{.\pi}$ VE $V_y = \frac{\mathbf{200}}{\mathbf{3}}\boldsymbol{.\pi}$ VE $V_x : V_y = 80 : 200 = 2 : 3$